

Game Theory and Applications Volume 4 (1998)

On a Partial Information Multiple Selection Problem	1–10
<i>K. Ano</i>	

Abstract

We consider a variation of the so-called partial information optimal selection problem. The buyer desires to buy an item at as low a price as possible based on a finite sequence of price quotations obtained sequentially from various sellers. After each price quotation is received, the buyer must decide either to select the price or not. It is assumed that the buyer is allowed to make at most $m(\geq 1)$ selections. The buyer can hold at most m prices and can buy at the lowest price among m prices with the objective of maximizing the probability of buying the good at the lowest price among all of N prices. It is understood that when the distribution of price quotations is completely known, the optimal rule is the one-stage look-ahead rule for both the single selection and the multiple selection problem. We show in this article that the one-stage look-ahead stopping rule for the multiple selection problem is still optimal in some cases where the price distribution has unknown parameter(s) and the buyer's prior on this parameter undergoes Bayesian updating as successive prices are received. From the point of view of a Game theory, this is regarded as an one person game problem.

The Normalized Banzhaf Value and the Banzhaf Share Function	11–31
<i>R. van den Brink and G. van den Laan</i>	

Abstract

A *cooperative game with transferable utilities* –or simply a TU-game– describes a situation in which players can obtain certain payoffs by cooperation. A *value function* for these games is a function which assigns to every such a game a distribution of payoffs over the players in the game. A famous solution concept for TU-games is the *Banzhaf value*. This Banzhaf value is not *efficient*, i.e., in general it does not distribute the payoff that can be obtained by the ‘grand coalition’ consisting of all players cooperating together.

In this paper we consider the *normalized Banzhaf value* which distributes the payoff that can be obtained by the ‘grand coalition’ proportional to the Banzhaf values of the players. This value does not satisfy certain axioms underlying the Banzhaf value. In this paper we discuss some characterizations of the normalized Banzhaf value and compare these with other solution concepts such as, for example, the (non-normalized) Banzhaf value and the *Shapley value*.

Another approach to analyze efficient value functions is to consider *share functions* being functions which assign to every player in a TU-game its share in the worth of the ‘grand coalition’. We discuss the characterization of a class of such share functions containing the Banzhaf and Shapley share functions.

Finally, we generalize the concept of the *potential function* of a game as introduced by Hart and Mas-Colell to a class of potential functions and characterize any element of the class of share functions by the normalized marginal function of the corresponding potential function.

Potential and Consistency for Semivalues of Finite Cooperative TU Games	32–44
<i>I. Dragan</i>	

Abstract

In this paper, the Dubey/Neyman/Weber semivalues for finite cooperative TU games are considered. The Power Game for semivalues as an extension of the Power Game for the Banzhaf value is introduced. The Power Game in terms of the coalitional form of the game is given. A potential is defined. Furthermore, it is shown that a semivalue is the Shapley value of the Power Game, and a recursive definition of the semivalues using the potential is given. The characterization of semivalue by means of consistency and weighted standardness is proved.

The Greedy Bankruptcy Game: an Alternative Game Theoretic Analysis of a Bankruptcy Problem	45–61
<i>T.S.H. Driessen</i>	

Abstract

This paper is devoted to researching a general bankruptcy problem from a game theoretic point of view, where the introduced game differs from the usual bankruptcy game. The introduced game is a TU game for which the core and the prekernel are analyzed. A long development is devoted to a study of the duality between TU games and indirect functions.

Game Problems on Rotation Surfaces	62–74
<i>N. Hovakimyan and L. Harutunian</i>	

Abstract

We consider differential games with simple motion of two points P (pursuer) and E (evader) on the surface of one sheet of double-sheeted two-dimensional rotation hyperboloid. We analyse geometrical properties of primary solution trajectories: geodesics and optimal phase structure. We also focus on the limit case of the hyperboloid, the two-dimensional cone, consider games of pursuit and approach and describe computer simulation results for this case.

Representation Games	75–86
<i>W. Kerby and F. Göbeler</i>	

Abstract

Here we present a short theory of sum games and value functions on sum games. We feel this theory is adequate to describe the power distribution of the members of such a legislative body as determined by the voting system itself. Any economic, military, political, social or cultural factors which may influence the decision making process are ignored. The distribution of voting power as measured by Shapley values is computed for the UN as it is presently structured. Also a generalization of multilinear extensions is described which leads to a general construction method for symmetric linear value functions. The classical Shapley and Banzhaf values are constructable by this method.

On Probabilities of Pure Strategy n-Tuples in Non-Cooperative Games	87–93
<i>V.L. Kreps and N.N. Vorob'ev</i>	

Abstract

Players mixed strategies taken as random trials with outcomes from the pure strategy set are supposed in the non-cooperative game theory to be stochastically independent. Stochastic independence is a quantitative relationship between probabilities of outcomes and probabilities of their combination (the latter are the products of the former).

The authors suggest other functional expressions for probabilities of combination of outcomes (other distributions over the set of pure strategy n -tuples) to be referred to below as types of dependence. It is supposed that probabilities of combinations of outcomes (probabilities of pure strategy n -tuples) are functions of probabilities of outcomes (components of the mixed strategies corresponding to these outcomes).

The purpose of this paper is to analyze the possibility of construction of the type of dependence of large dimension via the type of dependence of the smaller ones, in particular of dimension two. It is proved that reasonable suppositions pertaining to the thus constructed type of dependence result in the unique type of dependence that is the stochastic independence.

Strong $(N - 1)$-Equilibrium. Concepts and Axiomatic Characterizations	94–102
<i>D.V. Kuzutin</i>	

Abstract

Using a specific approach to the coalition consistency analysis in n -person strategic games we offer new Nash equilibria refinements: strong $(n - 1)$ -equilibria and strictly strong $(n - 1)$ -equilibria. The relation between these solutions and other coalition stable optimality principles (strong and strictly strong Nash equilibrium, coalition proof Nash equilibrium, semistrict Nash equilibrium) is established. We give the axiomatization of strong $(n - 1)$ -equilibrium for closed classes of strategic games in terms of consistency, one-person rationality, suitable variants of converse consistency and Pareto-optimality axiom and others.

An Optimal Stopping of Random Walks Game with Reflection	103–109
<i>V.V. Mazalov and E.A. Kochetov</i>	

Abstract

Consider a game of two players determined by random walks of the following form. Let x_n and y_n be independent symmetric random walks over the set $E_0 = \{0, 1, \dots, k\}$, beginning in state a and being absorbed at the ends with probability p and with probability $q = 1 - p$ reflecting from the state 0 and k into states 1 and $k - 1$, respectively. Players I and II observe the random walks x_n and y_n and stop them at some random time τ and σ , respectively.

These random variables τ and σ , which we call the stopping times are the players' strategies. If $x_\tau > y_\sigma$, Player I wins and if $x_\tau < y_\sigma$, Player II wins. But if $x_\tau = y_\sigma$ a draw is declared. There is no information about the opponent's behavior at a player's disposal. Here we find the equilibrium strategies for arbitrary p .

Note on the Core and Quasicores of Cooperative Games 110–120

S. Peckersky

Abstract

n -person cooperative and cooperative fuzzy games with sidepayments are considered. We consider the class Γ of superlinear (i.e. positively homogeneous and concave) fuzzy games and impose a set of axioms which defines a set-valued solution on Γ , the core being the unique solution satisfying the set of axioms. As a corollary of the corresponding uniqueness theorem we derive the analogous set of axioms for standard cooperative games. This set of axioms is in a sense intermediate with respect to Peleg's and Keiding's sets of axioms for the core. Finally, we define the notion of quasicores for a game and discuss some properties of this solution, which coincides with the Weber set for standard games, but appears in quite different context.

A Unique Nash Solution for the Games with

Perfect Information 121–129

L.A. Petrosjan

Abstract

An extensive form game is said to have perfect information if the following two conditions are satisfied: there are no simultaneous moves, and at each decision point it is known which choices have previously been made. In games with perfect information the subgame perfect equilibria can be found by dynamic programming, i.e. by inductively working backwards in the game tree. But even in this case one can find a wide class of Nash equilibria with different payoffs for the players. We propose the refinement of the Nash equilibrium concept based upon the preferences of the players, giving the possibility of determining the unique Nash equilibrium in the sense of the players' payoffs. The approach is used for the calculation of the unique Nash equilibrium in a differential "lifeline game" of pursuit with one pursuer and two evaders.

Nash-Hurwitz Equilibrium for Non-Cooperative Games	130–141
<i>L.V. Smirnova</i>	

Abstract

Non-cooperative n -person games with vector payoffs and uncertainty implying fuzziness in payoff functions are considered. A generalized equilibrium concept, called Nash-Hurwitz equilibrium, is introduced. For games with scalarized payoff functions equilibrium existence theorems are given for pure and mixed strategies cases. Under the same conditions, the existence of a Slater-maximal Nash-Hurwitz equilibrium is also shown. In addition to the above conditions, if the payoff functions are concave, the paper further shows the existence of a guaranteed Nash-Hurwitz equilibrium in pure strategies. In a two-person game with fuzzy linear quadratic payoffs an equilibrium is found.

Unimprovable Arbitration Solution Under Uncertainty	142–151
<i>K.S. Vaisman and S.S. Mishin</i>	

Abstract

A two-person bargaining problem under uncertainty without side payments is considered. It is presupposed, that the players follow the bargaining model by Nash and take into account a possibility of realization of any uncertainty. On the basis of the analog of vector saddle point arbitration NashSlater solution of the game is defined, and it is shown by example, that the set of such solutions is internally unstable. The unimprovable guaranteeing arbitration solution of Nash for the game as an internally stable solution is introduced; its properties and existence conditions are studied.

Continuous Take-Away Games	152–154
<i>S.V. Vinnichenko</i>	

Abstract

Epp and Ferguson [1] give a solution to take-away games. In such games a positive number m and a nondecreasing function $f(n)$ are given. The first player on his initial move can remove from a pile a positive number of counters n_1 , which does not exceed m . On the next move, the second player can remove from the pile a positive number of counters n_2 , which does not exceed $f(n_1)$, and so on. The player removing the last counter wins the game. This game is a variant of the game NIM (see, for example, [2]). In this paper similar continuous games are considered.

The Proper Sharley Value.....	155–159
<i>N.N. Vorob'ev and A.V. Liapounov</i>	

Abstract

Kalai and Samet and independently of them Diubin have proposed to consider the weighted Shapley Value as a solution to a class of cooperative games with a finite set of players, which depends on both the characteristic function and a given measure on the set of players. In this paper for a special class TU games, containing the monotone ones, the notion of the Proper Shapley Value (PSV) is introduced as a fixed point of a Shapley mapping, which transforms an imputation to a more fair imputation. Thus the PSV is a solution, which depends only on the characteristic function. It turns out that the PSV has an extremal property: it maximizes the production function of Kobb-Douglas types on the set of the imputation.

On Communication in Cooperation Games: a Survey.....	160–173
<i>O. Voshtina</i>	

Abstract

This survey gives generalizations of the Shapley value for the case when some of the coalitions, including the grand coalition are not possible. For a graph restricted game the Myerson value is given. It is shown that this value can be interpreted in a manner similar to the Shapley value. For a game in partition function form the definition of the position value and an extension of the Myerson are considered. Also another extension of the Shapley value is given axiomatically.

Communication Can Ease Prisoners' Dilemma.....	174–182
<i>J. Watanabe</i>	

Abstract

The purpose of this paper is to propose an extended game with a preplay negotiation in order to ease a two person prisoners' dilemma. That is, we present an explicit procedure that yields an efficient solution as its outcome.

The extended game is divided into the two stages: the preplay negotiation stage and the action decision stage. At the beginning of the preplay negotiation stage, the players face a 2×2 prisoners' dilemma game. Each player proposes a third strategy available for himself (or herself). A proposed strategy must be feasible in the sense that it signifies voluntary transfer from one player to the other or free disposal of some his (or her) payoff. Then, either a 2×2 , 3×2 , or 3×3 game is chosen depending upon a conclusion of the negotiation. In the action decision stage, the induced game created in the previous stage is given. And then the players make nonbinding communication, coordinate their strategies as long as they are self-enforcing, and obtain payoffs according to their correlated strategy.

It is shown that for some class of prisoners' dilemma games efficient payoffs than the noncooperative payoff of the original game can be obtained in the extended game.

We introduce a two person prisoners' dilemma game and briefly discuss the equilibrium in the game. An extended game is presented using an example and the result is proved formally.

Abstract

In this paper we consider the problem of existence of the core from linear programming point of view and treat the concept of subcore. One of the most important advantages of the linear programming approach is that the subcore may be represented in a simple analytical form. In addition, we can easily check the necessary and sufficient conditions for all one point solutions like Shapley value, Banzhaf value, egalitarian nonseparable contribution value (ENSC value), etc., to be selectors of the subcore. We also discuss in this paper the consistency property of the subcore in reduced games due to Davis-Maschler, Moulin and Funaki. A general consistency property of a solution is described as follows. Choose any payoff vector in a solution set for some game. Suppose that some coalition S want to renegotiate payoff distribution among his members and may cooperate with other members outside S paying for them agreed payoff. Such a process is described by a reduced game. The solution is called consistent if it recommends the same payoff distribution for the reduced game as initially. A consistent solution provides a stability of an agreement against wishes of any coalition to renegotiate agreed upon payoff distribution. In the last section we introduce and treat the grand subcore which is a generalization of the subcore. We proved that subcore and grand subcore are S -consistent in DM-, M-, and SIM- reduced games a special SC-condition holds.